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Propagation Characteristics of a Partially Filled Cylindrical Waveguide for Light Beam Modulation

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***Abstract*—Propagation characteristics of a cylindrical waveguide partially filled with a cylindrical dielectric light modulation material is analyzed and numerical computation is performed for a few typical cases. This type of traveling-wave structure supports a TE or TM mode, and therefore is useful for light beam modulation applications requiring a longitudinal magnetic or electric field. Numerical analysis indicates that for the TM case, a broadband region occurs near a crossover point and broadband synchronization between the modulating microwave and the parallel-launched light beam can be obtained by a suitable choice of the dielectric medium surrounding the central dielectric.**

Introduction

BROADBAND microwave modulation of a coherent light beam calls for a microwave traveling-wave structure which provides 1) a broad nondispersive region in the ω-β relation and 2) matching between the phase velocity of the microwave signal and the component velocity of light along the microwave traveling direction [1], [2]. To satisfy the first requirement one of the approaches is to utilize a microwave circuit possessing TEM wave-like propagating properties. To satisfy the second requirement, it is generally necessary to zigzag the optical path along the waveguide since for most electrooptic or magnetooptic materials the index of refraction is much less than the square root of the dielectric constant at the modulation frequency. Thus far, only one simple structure has been proposed [2] in which a dielectric material is partially filled between two parallel plates, and such a circuit resembles a TEM waveguide when the dielectric constant of the electrooptic material approaches that of the surrounding dielectric medium. It is this feature of partial filling that provides two distinct nondispersive regions in the ω-β diagram as analyzed by Kaminow and Liu [2]. Transverse electric fields are employed for modulation in their circuit. Based on this circuit DiDomenico and Anderson have discussed extensively the various features that influence the modulation performance [3].

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Whereas this type of TEM waveguide is simple and useful, it is, however, desirable in many instances to modulate a light beam by a longitudinal field. Modulation by the magnetooptic Faraday effects for example, requires such a field. It is therefore important to study microwave propagation circuits supporting a TE or TM mode. In this paper, we shall investigate the propagation characteristics of a cylindrical waveguide partially filled with a concentric dielectric cylinder as shown in Fig. 1. Since the metallic boundary is closed, this circuit will support a TM or TE wave, and longitudinal electric or magnetic fields are obtained. Theoretical analysis as well as computer results for a few typical cases are presented in the following.

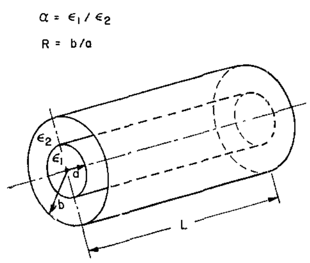

Fig. 1. Geometrical configuration of the partially filled cylindrical waveguide.

Circuit Analysis

The wave equations appropriate for the partially filled cylindrical waveguide shown in Fig. 1 are given by,

$$\nabla_{r,\phi}^2 E_1 = -(k_1^2 - \beta^2)E_1 \tag{1}$$

$$\nabla_{r,\phi}^2 E_2 = -(k_2^2 - \beta^2)E_2 \tag{2}$$

where

$$k_1 = \omega\sqrt{\mu_1\epsilon_1} \quad \text{and} \quad k_2 = \omega\sqrt{\mu_2\epsilon_2}. \tag{3}$$

The subscripts 1 and 2 refer to field parameters in region 1 (inside the central dielectric cylinder) and region 2 (between the central dielectric cylinder and the metallic boundary), respectively. The time and space variation of $e^{j(\omega t-\beta z)}$ is assumed and cylindrical coordinates are used. Similar equations hold for the magnetic fields H_1 and H_2.

The general field amplitudes in region 1 and region 2 can be expressed by Bessel functions and trigonometric functions in cylindrical coordinates [4]. The z-component electric and magnetic fields are:

$$\text{Region 1}\quad \text{TM}\quad E_{z1} = AJ_n(k_{c1}r)\begin{matrix}\cos n\phi\\ \cdot\\ \sin n\phi\end{matrix} \tag{4}$$

$$\text{TE}\quad H_{z1} = BJ_n(k_{c1}r)\begin{matrix}\cos n\phi\\ \cdot\\ \sin n\phi\end{matrix} \tag{5}$$

$$\text{Region 2}\quad \text{TM}\quad E_{z2} = [CJ_n(k_{c2}r) + DN_n(k_{c2}r)]\begin{matrix}\cos n\phi\\ \cdot\\ \sin n\phi\end{matrix} \tag{6}$$

$$\text{TE}\quad H_{z2} = [EJ_n(k_{c2}r) + FN_n(k_{c2}r)]\begin{matrix}\cos n\phi\\ \cdot\\ \sin n\phi\end{matrix}\cdot \tag{7}$$

A through F are amplitude constants, and

$$k_{c1}^2 = -\beta^2 + k_1^2 \tag{8}$$

$$k_{c2}^2 = -\beta^2 + k_2^2. \tag{9a}$$

Only the case $n=0$ will be discussed in the following analysis since fields having no angular variation are most appropriate for light modulation applications. Hybrid modes are therefore not considered in this analysis.

When the microwave phase velocity in the waveguide is slower than the free space phase velocity in a medium of dielectric constant ϵ_2, it is convenient to introduce

$$k_{c2}'^2 = -k_{c2}^2 = \beta^2 - k_2^2. \tag{9b}$$

Such choice always gives real k_{c2} or $k_{c2'}$. In general (9b) is applied for frequencies higher than cutoff while (9a) applies near cutoff.

By means of (4) through (9), one arrives at the various field components shown in Table I.

TABLE I
Nonzero Field Components for TE and TM Modes

Mode	Nonzero Field Components	Region I	Region II
	E_z	$AJ_0(k_{c1}r)$	$CI_0(k_{c2}'r) + DK_0(k_{c2}'r)$
TM	E_r	$-\dfrac{A\beta}{k_{c1}}J_0'(k_{c1}r)$	$\dfrac{\beta}{k_{c2}'}[CI_0'(k_{c2}'r) + DK_0'(k_{c2}'r)]$
	H_ϕ	$-\dfrac{Aj\omega\epsilon_1}{k_{c1}}J_0'(k_{c1}r)$	$\dfrac{j\omega\epsilon_2}{k_{c2}'}[CI_0'(k_{c2}'r) + DK_0'(k_{c2}'r)]$
	H_z	$BJ_0(k_{c1}r)$	$EI_0(k_{c2}'r) + FK_0(k_{c2}'r)$
TE	H_r	$-\dfrac{B\beta}{k_{c1}}J_0'(k_{c1}r)$	$\dfrac{\beta}{k_{c2}'}[EI_0'(k_{c2}'r) + FK_0'(k_{c2}'r)]$
	E_ϕ	$\dfrac{Bj\omega\mu_1}{k_{c1}}J_0'(k_{c1}r)$	$-\dfrac{j\omega\mu_2}{k_{c2}'}[EI_0'(k_{c2}'r) + FK_0'(k_{c2}'r)]$

For TM mode, the boundary conditions lead to the following determinantal equation:

$$J_0(p)[I_0(qR)K_1(q) + I_1(q)K_0(qR)] - \frac{\alpha q}{p}J_1(p)[I_0(q)K_0(qR) - I_0(qR)K_0(q)] = 0 \tag{10}$$

where

$$p = k_{c1}a, \quad q = k_{c2}'a, \quad R = \frac{b}{a} \quad \text{and} \quad \alpha = \frac{\epsilon_1}{\epsilon_2}\cdot \tag{11}$$

For the TE mode, the determinantal equation is given by

$$I_0(p)[I_1(q)K_1(qR) - I_1(qR)K_1(q)] - \frac{qJ_1(p)}{p}[I_0(q)K_1(qR) + I_1(qR)K_0(q)] = 0. \tag{12}$$

TABLE II
EQUATIONS FOR OBTAINING PROPAGATION CHARACTERISTICS

L.H.S.	R.H.S. ($N > \sqrt{\epsilon_2}$)	R.H.S. ($N < \sqrt{\epsilon_2}$)	Mode
$p \dfrac{J_0(p)}{J_1(p)} =$	$\alpha q \dfrac{I_0(q)K_0(qR) - I_0(qR)K_0(q)}{I_0(qR)K_1(q) + I_1(q)K_0(qR)}$	$\alpha q \dfrac{J_0(qR)N_0(q) - J_0(q)N_0(qR)}{J_0(qR)N_1(q) - J_1(q)N_0(qR)}$	TM
	$q \dfrac{I_0(q)K_1(qR) + I_1(qR)K_0(q)}{I_1(q)K_1(qR) - I_1(qR)K_1(q)}$	$q \dfrac{J_0(q)N_1(qR) - J_1(qR)N_0(q)}{J_1(q)N_1(qR) - J_1(qR)N_1(q)}$	TE
where $p = \nu \sqrt{1 - \left(\dfrac{N}{\sqrt{\epsilon_1}}\right)^2}$	$q = \nu \sqrt{\left(\dfrac{N}{\sqrt{\epsilon_1}}\right)^2 - \dfrac{1}{\alpha}}$	$q = \nu \sqrt{\dfrac{1}{\alpha} - \left(\dfrac{N}{\sqrt{\epsilon_1}}\right)^2}$	

The propagation characteristics are contained in these determinantal equations. We have assumed throughout that $\mu_1 = \mu_2 = 1$. Also dielectric losses are not considered.

For light modulation applications, it is convenient to introduce N and ν defined by

$$N = \frac{c}{v_{ph}} = \frac{c}{\omega}\beta, \tag{13}$$

the effective index of refraction for microwave propagation along the waveguide, and

$$\nu = k_1 a = \frac{2\pi}{\lambda_0}\epsilon_1 a, \tag{14}$$

a dimensionless parameter proportional to frequency.

Equations essential for obtaining the propagation characteristics are given in Table II. These equations are solved by numerical analysis on the computer for various typical cases. The results are plotted as curves of $N/\sqrt{\epsilon_1}$ vs. ν. They are given in Figs. 2 through 7. A variety of values for R and α have been used. The values of α computed correspond to practical light modulation materials, and are so indicated on the figures. Only the lowest order modes are computed since they are preferable for modulation.

TE Modes for Magnetooptic Modulation

Figures 2, 3, and 4 show three sets of TE propagation characteristics. We obtain the nondispersive high-frequency region as well as the low-frequency dispersive region, as is characterized by ordinary cylindrical wave guides. Only the high-frequency region is useful for broadband modulation. To match optical and microwave propagation for cumulative interaction one has to zigzag the optical path. To achieve this, a hollow cylindrical light beam can be made to diverge by a lens of appropriate focal length and thus be multiply reflected along the cylindrical dielectric rod boundary. If θ_0 is the angle that the light beam makes with the cylindrical axis, and the average microwave magnetic field in the z-direction is H_0, it can be shown that the Faraday rotation of the polarization vector of light is given by

$$\theta_F = V \cdot L \left[\frac{\sin\dfrac{\beta_s L}{2}}{\dfrac{\beta_s L}{2}}\right] H_0 \cos\left(\omega t - \frac{\beta_s L}{2}\right)\cos\theta_0$$

and

$$\beta_s = \frac{n\omega}{c\cos\theta_0} - \beta$$

where V is the Verdet constant, L the length of the modulator, ω the modulation angular frequency, β the microwave propagation constant, and n the optical index of refraction of the modulation medium. β_s is zero under the synchronism condition, giving maximum rotation of polarization. One can use an analyzer to convert a polarization modulated beam to an amplitude modulated beam.

It may seem that the radial magnetic field will affect the polarization modulation, but actually, under synchronism, the effects produced by the opposing radial magnetic fields will cancel each other as the light beam is traversing the dielectric rod. Another point to be mentioned is the mode separation. From our calculation, the separation between the cutoff points of adjacent TE modes corresponds approximately to $\Delta(k_1 a) = \pi$

$$\Delta f \cong \frac{c}{2\sqrt{\epsilon_1}a}.$$

If $a = 1$ cm, $\epsilon_1 = 4$, then $\Delta f \cong 7.5$ Gc/s. This separation is comfortable even for broadband applications.

TM Mode for Electrooptic Modulation

Three sets of TM mode propagation characteristics are shown in Figs. 5, 6, and 7. One observes that, besides the higher frequency nondispersive region, another

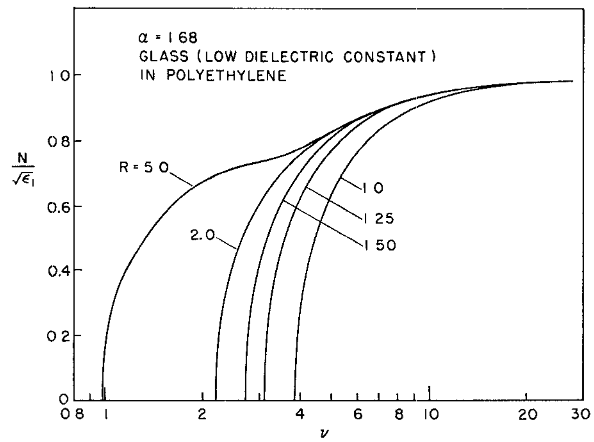

Fig. 2. TE mode propagation characteristics for $\alpha=1.68$.

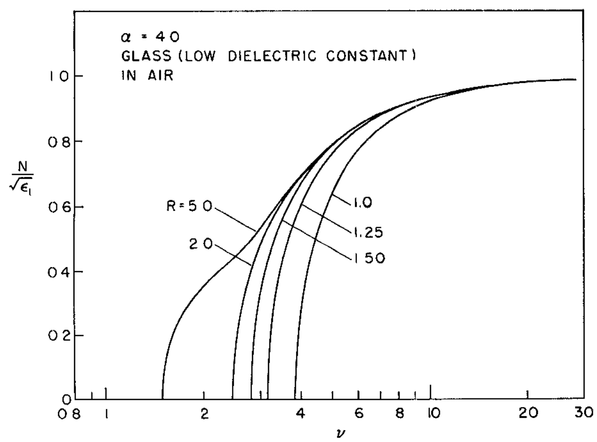

Fig. 3. TE mode propagation characteristics for $\alpha=4.0$.

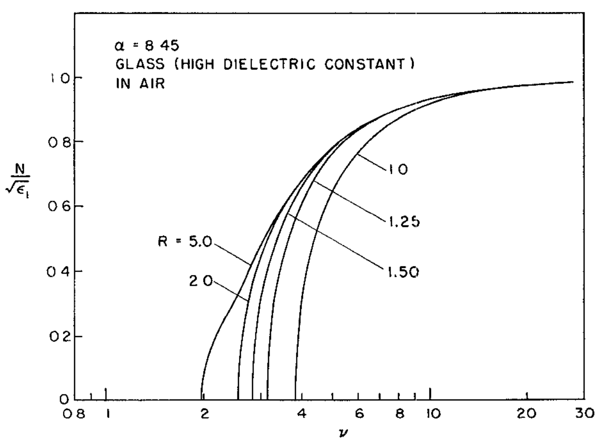

Fig. 4. TE mode propagation characteristics for $\alpha=8.45$.

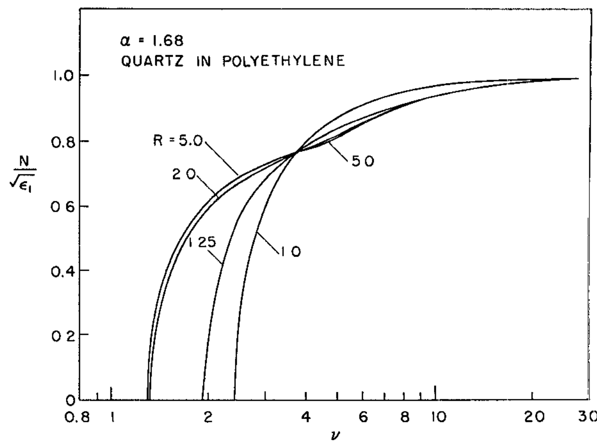

Fig. 5. TM mode propagation characteristics for $\alpha=1.68$.

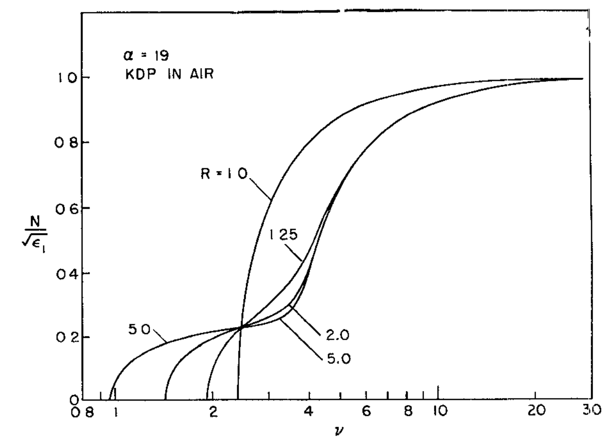

Fig. 6. TM mode propagation characteristics for $\alpha=19$.

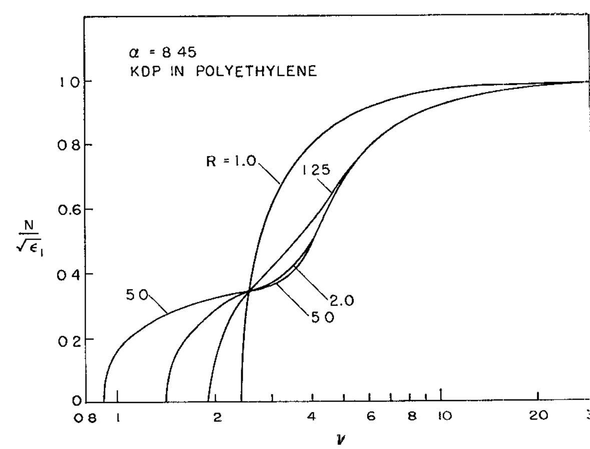

Fig. 7. TM mode propagation characteristics for $\alpha=8.45$.

plateau region gradually appears as the value of R is raised. This region occurs in the neighborhood of the crossover point for curves of various R values. As the ratio of dielectric constants increases, the flatness of the plateau becomes more pronounced. This second nondispersive region is attractive for broadband modulation in that one can place it up or down by adjusting the dielectric constant of the material surrounding the E-O crystal. Thus velocity matching between light and microwave is made possible when one launches the light beam along the waveguide axis.

As a matter of fact, the crossover occurs when N is equal to $\sqrt{\epsilon_2}$; therefore, to ensure broadband synchronism for an axially launched optical beam, one needs to choose a supporting dielectric such that $\sqrt{\epsilon_2}$ is equal to the index of refraction of the E-O crystal. For instance, the index of refraction for KDP is 1.5, which is identical to $\sqrt{\epsilon_2}$ of polyethylene ($\epsilon_2=2.25$). Therefore, in this cylindrical waveguide geometry, parallel launching of light and microwave is made possible by using a KDP central cylinder if it is surrounded by polyethylene. This feature relieves the many undesirable problems when a zigzag optical path is used.

Microwave propagation in a waveguide can be considered as plane waves reflected between opposing metallic boundaries (strictly true for rectangular waveguides only). It is interesting to note that the crossover point for the TM characteristics corresponds to the total internal reflection angle between ϵ_1 and ϵ_2. Were it not for the existence of the outer metallic boundary, the crossover point would have corresponded to the cutoff point of the circuit because the electromagnetic energy is no longer confined within the central dielectric at frequencies below this point. However, with the metallic boundary, the microwave energy leaking out of the dielectric rod will still be reflected back; therefore, wave propagation is still possible below this point. It is for this reason that this crossover point should be called the critical point instead of the cutoff point [5], [6] since it does not bear the conventional meaning of cutoff. The true cutoff point for this partially filled cylindrical waveguide occurs at a lower frequency, where the characteristic curve intercepts the ν axis. It can be shown that, for a rectangular waveguide of width $2b$ with a slab of dielectric of thickness $2a$ centered in the middle, the cutoff can be ascertained by requiring that the microwave phase difference between the two metallic boundaries be π radians, that is

$$\frac{4\pi}{\lambda}\left(\sqrt{\epsilon_2}\,(b-a)+\sqrt{\epsilon_1}\,a\right)=\pi.$$

For the partially filled cylindrical waveguide case, an approximate relation can be derived from this, i.e.,

$$k_1 a\left[\frac{R-1}{\sqrt{\alpha}}+1\right]=\frac{\pi}{2}\cdot$$

It turns out that cutoff points obtained from the simple formula do agree with computer results fairly well.

A possible modulation scheme for TM mode interaction would be to use a polarizer, followed by a quarter-wave plate, then followed by this traveling-wave cylindrical modulation circuit, and then an analyzer at 45° to the polarizer, as has been discussed by other authors [3].

Conclusion

The propagation characteristics of a cylindrical waveguide partially filled with a central cylinder of light modulation material are analyzed and computer results presented. The intent of this study is to obtain necessary information on the properties of this waveguide system for light beam modulation applications. Broadband modulation of a light beam requires a traveling-wave circuit which is dispersionless over a range of frequencies. The only modulation circuit analyzed up to date is a partially filled parallel plate circuit which supports a TEM-like mode. For efficient modulation of a light beam, it is often desirable to utilize the longitudinal electromagnetic field; therefore, circuits supporting TE or TM modes are required. The waveguide system studied here supports these modes, and possesses many of the features desirable for broadband modulation purpose. Various values of the geometrical parameters as well as typical dielectric modulation material parameters are used in this analysis. The problem of dielectric losses and boundary wall losses are not considered. Our results show that for magnetooptic modulation one can make use of the longitudinal magnetic field of the TE mode, but zigzagging of a circular hollow light beam is required. For electrooptic modulation one can make use of the longitudinal electric field of the TM mode and, if the supporting dielectric is properly chosen, axial launching of a light beam is possible while synchronism of microwave and light velocities is still maintained over a broad frequency range.

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